A Quantitative Implementation of Black Litterman Model

– An Empirical Study in the IT Sector

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Abstract *Motivated to examine the effectiveness of the Black-Litterman model and find a more practical model for portfolio optimization, we extend our previous paper on the comparison of three risk measures: variance, expected shortfall and factor model covariance. We collected investors' views from analysts' reports and incorporated them into the market views to obtain our posterior returns and conducted the Markowitz portfolio optimization with the risk measures. By tuning the constraints and parameters over the period of 2010 to 2014, we showed that the implementation of the BL model helps enhance the portfolio performance and the outperformance is consistent throughout all of the three risk measures for 2014-2018. Therefore, our paper makes a contribution by providing a quantitative approach of obtaining investor's views. We found an empirical evidence of the effectiveness of the Black-Litterman model, which advocates for the application of the Black-Litterman model for portfolio management in the financial industry.*

1 Introduction

In the previous paper, we were motivated by the uncertainty in the effectiveness of different risk measures across time and sectors. Hence, we studied variance, conditional value at risk (CVaR) and factor model covariance, and applied them in the mean-risk models for portfolio optimization. By examining the best-performing risk measure for both recession periods and recovery periods, we show that CVaR with restrictive constraints on asset weight holdings may be a better option for preventing huge losses during recession periods for companies in the technology sector.

Markowitz proposed the use of simple variance from historical returns as a risk measure to find an optimal portfolio, but unfortunately, the model has some shortcomings, such as its tendency to assign extreme portfolio weights or its high sensitivity to inputs. This proved to negatively impact our portfolio allocation, as shown by our previous results, in which our portfolios underperformed the benchmark portfolio, especially when optimizing the portfolio under more lax constraints. We observed that this occurred not just during times of crises, but in almost all the time periods we examined.

One way to address these issues is by using the Black-Litterman framework. Markowitz is sensitive to input parameters, which means that a small change in the expected return of one stock will drastically change the weights on the stocks. Therefore, using Black-Litterman to obtain the expected return vector will provide better and more stable input estimates and in turn stabilize the weights assigned to the assets in the portfolio. This will prevent the optimization model from making drastic, counter-intuitive shifts in portfolio weights, which was what we had seen in our previous paper. Additionally, since the expected returns generated by the Black-Litterman model include analyst consensus, they are also more forward-looking and reliant on historical data, which can be especially useful during times of turmoil.

In this paper, we continue studying the performance of risk measure but instead of simply using the historical expected return in the mean-variance optimization, we add an additional step of using the Black-Litterman framework to generate a different expected return vector that takes analyst views as well as our confidence in those views into account.

The paper is structured as follows. In Section 2, we survey the development of the Black Litterman model and discusses our potential contributions to the literature. Section 4 contains a description of the data and how we collect and process the data. Section 5 and Section 6 detail the constructions of posterior views in the BL model and the mean-risk models with three different risk measures in addition to the tuning process. Section 7 reports and explains the empirical result of optimization models for each risk measure with the BL model over the period of 2014-2018. Section 8 summarizes our conclusions from the results and outlines some possible extensions.

2 Literature Review

As shown in our last project, Markowitz portfolio optimization is sensitive to input parameters and creates highly concentrated portfolios if there is no constraint on the portfolio concentration. Suggested by Best and Grauer (1991) and Chopra and Ziemba (2013), marginal changes in expected returns can lead to large variations of the optimal weights, while the weights are less sensitive to variances and covariances. To mitigate the drawbacks of the traditional Markowitz portfolio optimization, Black and Litterman (1992) in Goldman-Sachs developed Black-Litterman (BL) model that combines the expected returns with investors' prior views. Lee (2000) showed that the BL model largely mitigates the problem of estimation error-maximization in the MV optimization by spreading errors throughout expected returns.

Due to the intuitive portfolio composition, the BL model has been accepted in a considerate amount of academic studies. However, it is not widely applied as expected since Black and Litterman (1992) did not include instructions on constructing the covariance matrix of the views. A number of papers attempted to clarify on the derivation of the estimates and provide step-by-step recipes and guidance on how to derive the uncertainty of views and the investor views (Meucci, 2010; Satchell and Scowcroft, 2000). He and Litterman (1999) provided examples to show the difference between the BL optimization process rather than the mathematics behind them. Mankert (2006) applied mathematical and behavioural approaches to the BL model to generate better knowledge and examples.

Further extensions and improvements of the BL model have been suggested in the literature. One of the strong assumptions of the Black-Litterman model is the normality of the random return vector. Satchell and Scowcroft (2000) extended the approach of the BL model to the second moments of distribution. Moreover, Meucci (2006) extended to the non-normal distribution on returns and considers heavy tails for other types of asset classes such as hedge funds and derivatives. Martellini and Ziemann (2007) extend the BL model by considering 4-moment-CAPM model instead of the standard CAPM model for the estimation of the market neutral implied views.

In addition, literature has explored on additional ways to establish investors' views. Fabozzi et al. (2006) combined a cross-sectional momentum strategy with market equilibrium using the BL model in the mean-variance framework. Jones et al. (2007) adopted Fama French factor models and the momentum factor to obtain views. The BL model has been generalized using an interpretation as an inverse optimization problem so mean and covariance of the returns are determined through a conic program.

Studies have explained ways to assess views but provided relatively limited empirical evidence that the BL model generates a superior portfolio performance relative to MV portfolio and passive portfolio management. Therefore, our work to compare the BL model's posterior returns to the historical returns can effectively examine the performance of BL model. Meanwhile, based on our research, few studies have compared different risk measures based on the construction of the BL model. Hence, by combining our first project on different risk measure comparison, we can contribute to the literature by analyzing and finding the best performing combination of risk measure and return estimates.

3 Black-Litterman Model

With the Black-Litterman model, we can obtain a conditional distribution of returns by combining the market information and investors' views. It is assumed that investors have prior views regarding the returns of portfolios specified in matrix A.

The following is a list of notations we use in the BL model for reference, which is consistent with the lecture slides.

μ'' = the expected return vector that incorporates market and investors' views
μ' = the expected returns of views
r = a vector of asset returns
T = number of samples
V = non-singular covariance matrix of excess ret computed from historical returns
Σ = non-singular covariance matrix of historical expected excess returns (sample average)
b = forecast return expectations
U = covariance matrix of the views
W = covariance matrix of the posterior view

For definiteness, let $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r^{(t)}$ be the average of T sample returns, so it is based on returns since the trend started. Assumed that the returns r^t follow a Gaussian distribution $N(\mu^*, V)$. Hence, $\hat{\mu}$ follows $N(\mu^*, \Sigma)$, where $\Sigma = \frac{1}{T}V$. The density function is

$$f_\mu(\hat{\mu}) = \frac{\exp(-\frac{1}{2}(\hat{\mu}-\mu)^T\Sigma^{-1}(\hat{\mu}-\mu))}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \qquad (1)$$

In the Bayesian theory, $P(H|E) = \frac{P(E|H)P(H)}{P(E)}$ so $P(H|E)$ is the conditional probability of H given E equals the probability of a correct view given prior information about historical returns. Hence, in the Bayesian setting, the distribution of posterior view in BL model is $g(\theta)f_\theta(\overline{z})$, which is obtained by multiplying prior distribution by the likelihood function of specific observations.

3.1 Views as linear equations

Let A be the matrix whose row is α_i^T, which specifies a particular portfolio in which we have a belief and A is $m \times n$ matrix with m views and n stocks. So $\alpha_i^T \mu = b_i$ for i from 1 to m view, which is $A\mu = b$. Given that $A\mu$ follows a Gaussian distribution, our view $b \sim N(\overline{b}, U)$. Hence the density function is $b = \frac{1}{(2\pi)^{\frac{n}{2}}|U|^{\frac{1}{2}}}\exp(-\frac{1}{2}(\overline{b}-b)^T U^{-1}(\overline{b}-b))$. Substituting $A\mu$ for b and discarding the constant term, we obtain the prior distribution which is

$$g(\mu) = \exp(-\frac{1}{2}(A\mu-\overline{b})U^{-1}(A\mu-\overline{b})). \qquad (2)$$

Following the Bayesian method, for specific observation $\hat{\mu}'$ of $\hat{\mu}$, the posterior distribution is

$$\begin{aligned} g(\mu)f_\mu(\hat{\mu'}) &= \exp(-\frac{1}{2}(A\mu-\overline{b})U^{-1}(A\mu-\overline{b}) \\ &\qquad -\frac{1}{2}(\hat{\mu}'-\mu)^T\Sigma^{-1}(\hat{\mu}'-\mu)) \\ &= C\,\exp(-\frac{1}{2}(\mu-\mu'')^T W^{-1}(\mu-\mu'')) \end{aligned} \qquad (3)$$

where W is the covariance matrix of view b and

$$W = (A^T U^{-1} A + \Sigma^{-1})^{-1} \qquad (4)$$

$C = \exp(\overline{b}^T U^{-1}\overline{b} + ((\hat{\mu}')^T V^{-1}\hat{\mu}' - (\mu'')^T\mu'')$ and

$$\mu'' = W(A^T U^{-1}\overline{b} + \Sigma^{-1}\hat{\mu}') \qquad (5)$$

Since the posterior distribution is proportional to the Gaussian distribution with μ'' and covariance matrix W, we use μ'' as our returns in Markowitz optimization.

3.2 Covariance matrix of views

Since the covariance matrix for the portfolios is $m \times m$ matrix AVA^T, if the views are equally certain in each of the m views, uncertainty in the investors' views can be expressed as

$$U = \lambda A V A^T \qquad (6)$$

for some positive scalar λ. In Meucci (2009)'s book, $\lambda = \frac{1}{c} - 1$, and he shows that $c = \frac{1}{2}$ represents that the investors are trusted as much as the official market.

Based on Meucci (2010)'s work, the best guess of the portfolio's return during the holding period

$$\overline{b}_i = (\hat{\eta}')^T\alpha_i + \kappa_i\sqrt{\alpha_i^T V \alpha_i} \qquad (7)$$

where $\hat{\eta}'$ is the expected return vector computed from samples and k_i is a constant chosen by us. As suggested by Meucci (2010), the parameter $k_i \in \{\pm1, \pm2\}$, representing the magnitude of the investors' views.

The number of analysts' forecasts is linked to the view confidence matrix. Following Becker and Gürtler (2010)'s approach to obtaining the confidence matrix, we let the maximum number of analyst' forecasts that are given over the whole time period have a confidence probability of 100%. If there is no forecast of asset i at time t, the confidence probability would be 0%. Therefore, the confidence probability is based on the number of analysts' forecasts for each asset i at time t, in the range of 0% and 100%.

After calculating $\bar{b}$ and substituting U into W, we can obtain the uncertainty matrix from views and hence the expected return matrix μ'' for optimization.

4 Data Collection & Processing

Using the same set of stock prices dataset as in our first project, we have a total of 37 companies that are listed in the information technology sector of the S&P 500 index. The weekly factor returns data used in our macroeconomic factor model are obtained from Fama and French's website[1]. We then merged the weekly stock prices of those 37 companies with the Fama and French's 3 factors.

The views are manually collected from analyst recommendations on Bloomberg Terminal. For each stock, we recorded the majority consensus amongst different analysts on whether to buy, hold, or sell the stock. We also obtained the weekly 12-month target prices for each stock, which are then used to compute the size of the expected returns corresponding to the views. The target prices are set in relation to the spot price at the time of the recommendation in order to get the expected future return of our views.

Since the earliest analyst views available on Bloomberg Terminal are in July 2010, we tune the parameters and train our model from July 2010 to August 2014. Among the 210 weeks, we use the first 110 weeks to obtain historical covariance matrix V, start trading and rebalancing biweekly from week 110 to week 210; hence, the number of samples is 100 for our optimization. Then, we apply the same process and test the performance of the tuned model during the 2014-2018 period in the empirical analysis section.

5 BL Model Construction

We used analyst recommendations for the prior view. For each stock i, we compared the number of analyst recommendations and pick the majority. If a majority of analysts recommend 'buy', we assign the value 1 to $A_{i,i}$. If a majority of analysts recommend 'sell', we assign the value -1 to $A_{i,i}$. If a majority of analysts recommend 'hold' or if the number of analysts that recommend 'buy' is equal to the number of analysts that recommend 'sell', we assign the value 0.005 to $A_{i,i}$. Ideally, we would set the value to be 0, but instead, we chose the small positive number 0.005 to prevent the result from being a singular matrix. Note that A is a diagonal matrix.

Next, we compute the matrix λ using the formula $\lambda_{i,i} = \frac{1}{c_i} - 1$ where c_i is a positive scalar, as suggested by Meucci (2009). When $c \to 0$, the investors' views have no impact since there is an infinitely disperse distribution of views. When $c \to 1$, investors are trusted completely over the market since there is an infinitely peaked distribution of views. When $c = \frac{1}{2}$, investors are trusted as much as the official market. For each stock i, we used the proportion of analysts that agree with the majority consensus to represent our confidence in that view. We interpret c as the confidence probability such that it is between 0% and 100%. Hence, calculate c for each i such that

$$c_i = \frac{\text{Num. analysts agree with the consensus}}{\text{Total number of analysts}} \tag{8}$$

Given that there are cases where either denominator or numerator of c is 0, we decide to use a different formula to calculate the value of $\lambda_{i,i}$. The first case is when the numerator, the number of analysts that agree with our chosen consensus, is zero. This happens when half of the analysts recommend 'buy' and the other half of the analysts recommend 'sell' so there are no analysts that recommend 'hold'. In this case, to prevent singularity, we set the value 0.5 to $\lambda_{i,i}$ to represent our relative certainty in the view. The second case is when all analysts agree with our consensus. Under this

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

circumstance, we do not wish to express an absolute certainty in this view, so we set $\lambda_{i,i}$ to be 0.05 to represent a 5% uncertainty. Note that λ is a diagonal matrix.

Next, we compute the vector $\hat{\mu}'$. We first calculate the expected annual return using the 12-month target price and then convert it to weekly returns for each stock i using the formula below:

$$\hat{\mu}_i' = \frac{1}{52} . \frac{\text{12-month target price} - \text{today's price}}{\text{today's price}} \tag{9}$$

For parameter k in the Equation 7, according to Meucci (2010), $k \in \{-\beta, -\alpha, \alpha, \beta\}$ which defines "very bearish", "bearish", "bullish", "very bullish" views respectively. Typical choices for those parameters are $\alpha = 1$ and $\beta = 2$. Therefore, we choose the vector k such that

$$k_i = \begin{cases} 1 & \text{if } \hat{\mu}_i' > 0 \\ -1 & \text{if } \hat{\mu}_i' < 0 \\ 0.05 & \text{if } \hat{\mu}_i' = 0 \end{cases} \tag{10}$$

We chose the small positive number 0.05 to express some uncertainty in our views.

Using the values we obtained above, we calculate μ'' and use this as our expected return vector in our optimization models.

6 Portfolio Optimization Construction

For the risk-return portfolio optimization with the Black-Litterman model, we use the following objective function with respect to three different risk measures.

$$\text{maximize } \mu^T x \tag{11}$$

for x the proportion of investment in each asset and μ the expected return from each asset, which is μ'' obtained from the BL model.

We use the same set of constraints as in the first project which includes constraints on portfolio risk, transaction cost and its total cost, changes in allocation, no borrowing, portfolio holding limits, and turnover rate. For CVaR, the portfolio risk constraint would be $\ell + \frac{1}{1-\alpha} \sum_{w \in \Omega} p(w) \max\{loss(x,w) - \ell, 0\} \leq ES_{benchmark}$.

$$\begin{aligned} x^T V x &\leq allowable\_risk \cdot \sigma^2_{benchmark} \\ c \sum_{i=1}^{n} \left| y_i(t) \right| &\leq C \\ \sum_{i=1}^{n} (x(t)) + C &= 1 \\ x_i(t-1) &= y_i(t) + x_i(t) \\ x_0 &\geq 0 \\ \left| x_i(t) - \Delta \right| &\leq \delta \\ \sum_{i=1}^{n} \left| x_i(t) - x_i(t-1) \right| &\leq c \sum_{i=1}^{n} x_i(t-1) \end{aligned} \tag{12}$$

Since we use an equally weighted benchmark portfolio, we rebalance this portfolio by distributing a new level of wealth equally among all risky assets and riskless asset after each trading date. This portfolio's risk can be calculated using the following formula.

$$\sigma_{benchmark} = \frac{1}{n+1} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} V_{i,j} \right)^{\frac{1}{2}} \tag{13}$$

Additionally, given that we use an equally-weighted benchmark, in the holding limit constraint, we set $\delta = \frac{1}{n+1}$ where n is the number of stocks in our portfolio. This is to prevent the weights in our portfolio to deviate too much from the benchmark.

6.1 Risk Measures

Following our previous project, we continue utilizing the same three risk measures: variance, CVaR, Factor model covariance matrix.

$$\text{Var} = x^T V x \tag{14}$$

$$ES_\alpha(x) = \min_{\ell} \left(\ell + \frac{\sum_{w \in \Omega} p(w) \max\{loss(x,w) - \ell, 0\}}{1-\alpha} \right) \tag{15}$$

$$\text{Factor model covariance matrix} = BFB^T + \Delta \quad (16)$$

where x is the proportion of total wealth used to buy assets, V is the $n \times n$ historical return covariance matrix, $p(w)$ is the probability of scenario w, Ω is the finite set of scenarios, B is the matrix of factor loadings, F is the covariance matrix of f and Δ is the diagonal matrix whose (i,i) entry is $var(\epsilon_i)$.

6.2 Constraints and Parameter Tuning

To make our portfolio perform better during the optimization process, we want to tune some essential parameters and constraints in our optimization model on the training data set before we construct our Black-Litterman portfolios. We first tune three parameters on our test data set from July 2010 to August 2014.

rate_of_decay: we first choose the relative weight for one year earlier, r, meaning that the weight assigned to the sample one year earlier is r times the weight assigned to the most recent sample. Then we calculate the *rate_of_decay* by $1 - r^{sample\_frequency/52}$. We tune *rate_of_decay* by tuning r for $r \in [0.1, 0.5]$.

allowable_risk: the *allowable_risk* controls the highest allowable risk of the optimizing portfolios. We constrain our optimization model to be at most as risky as *allowable_risk* times the risk of the benchmark portfolio. In CVaR portfolio optimization, the allowable risk is used to calculate the $ES_{benchmark}$, which denotes as β. For the tuning range, *allowable_risk* $\in [1, 2]$.

α: when the *allowable_risk* increases, for CVaR portfolios, the pairing confidence level should decrease to protect the downside risk. So α is the pairing risk parameter for *allowable_risk* through the optimization of our CVaR portfolios. We tune *alpha* for $\alpha \in [0.9, 0.99]$.

To further improve the performance of our Black-Litterman portfolios, we also tune our optimization constraints and the parameters. Since the constraints on the risk of the portfolio, its transaction cost and its no borrowing nature are fixed, we only tune constraints on the holding limits. More specifically, the holding limit constraint limits the weight on each risky asset so that it does not deviate too much from the equal weight. Hence, we want to find the best weight intervals for risky assets. In other words, we are trying to tune Δ in the holding limit constraint since we set $\delta = \frac{1}{n+1}$. As we do not want the weight on each risk asset too large or too small, we make Δ changeable between $[\frac{0.5}{n+1}, \frac{2}{n+1}]$ where n is the number of risky assets.

As presented in Table 1, for each of our three risk measures, we find the best combination of these parameters through cross-validation on the performance of our portfolios with the specific risk measure. We figure out the following best combinations by comparing the final wealth, expected returns as well as Sharpe ratios. The rate of decay r is the highest for CVaR, which follows our expectation and suggests that more weight is put on the recent samples. The *allowable_risk* is also the highest for CVaR. The higher *allowable_risk* is, the higher β in CVaR is. Hence, the tuning output shows that a high value of CVaR risk β with a high confidence interval of 99% yields the best performance. The potential reason for this combination is that we are tuning our parameters during an economic booming time period (after the end of 2008 financial recession) and hence it is intuitive that the allowable risk is high for CVaR and the confidence level is high as well.

Table 1: Best Parameter Combinations for Portfolios with Different Risk Measures

	Variance	CVaR	Factor
r	0.2778	0.5000	0.4111
allowable_risk	1.4444	2.0000	1.5556
α	N/A	0.9900	N/A
Δ	$\frac{1}{n+1}$	$\frac{1}{n+1}$	$\frac{1}{n+1}$

This table reports the best parameter combinations for the three risk measures based on the highest expected returns and final wealth. α for the covariance-matrix portfolio and the factor model covariance-matrix portfolio is N/A as α is not needed during the optimization process of the other two portfolios with covariance and factor model covariance.

7 Empirical Analysis

After tuning our parameters and constraints on our training data set, we use the best combinations in the optimization of our Black-Litterman portfolios with the three risk measures. We construct

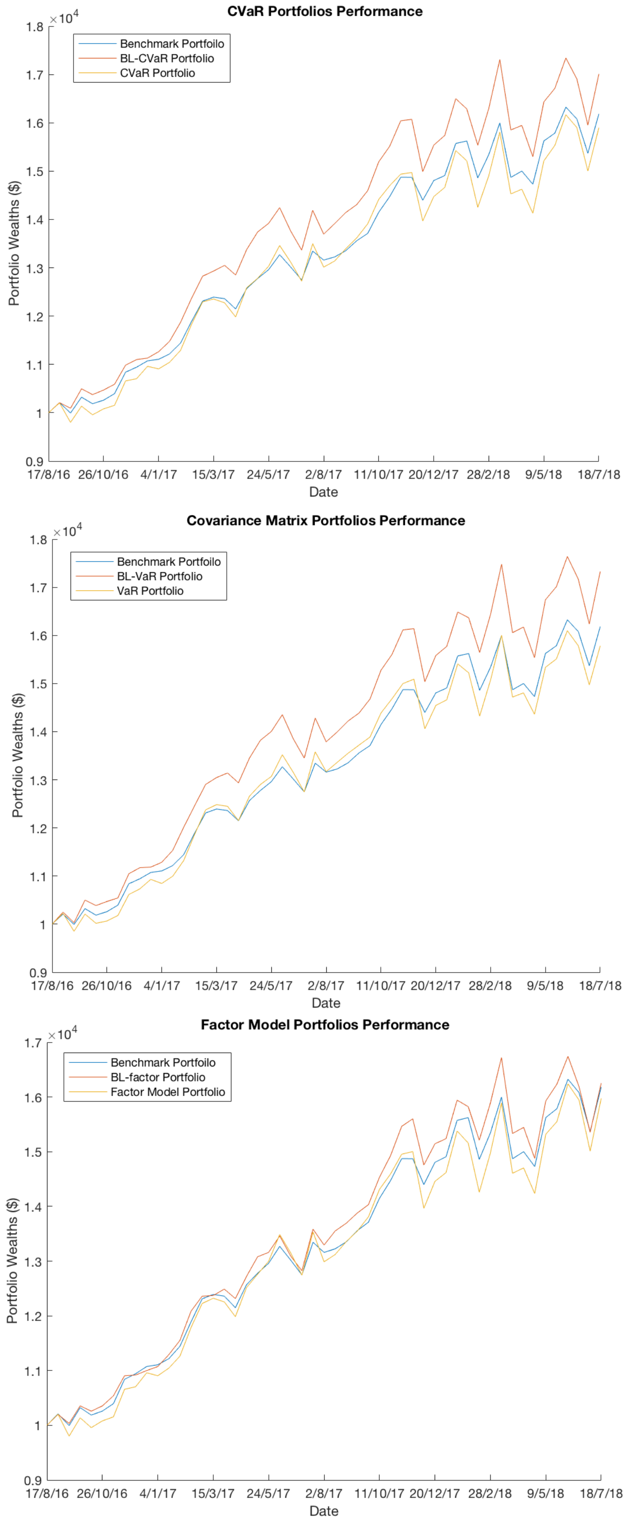

Figure 1: These figures show the wealth of CVaR, variance and factor model covariance portfolios during 2016-2018. This plot suggests that the performance of portfolio with Black-Litterman method and risk measures beats the performance of both the Markowitz portfolio with risk measures and the equal-weighted benchmark portfolio. Among all the risk measures, the implementation of the BL model and factor model covariance performs the best with the highest return.

and optimize our portfolios from 2014 to 2018 by using the first 110 weeks to obtain the historical covariance matrix, and trading and rebalancing biweekly during 2016-2018. Then we compare their performances with the performances of classical Markowitz portfolios to examine if the Black-Litterman method improves the returns of portfolios and results in better portfolio performance.

As we can see the performance of the CVaR portfolios on the top left of Figure 1, returns on the portfolio constructed with the help of Black-Litterman method is higher than the returns on the benchmark portfolio and the usual CVaR portfolio. Also, similar to the results in the previous project, the benchmark portfolio again outperforms the CVaR portfolio without the BL method. Therefore, using the return vector from the Black-Litterman method instead using the historical return vector in the optimization process helps enhance the performance and the return of CVaR portfolio.

Similarly for historical covariance matrix, as shown on the top right of Figure 1, with the combination of investors' views and market views, the returns on the portfolio outperforms the equal-weighted benchmark portfolio and the classical Markowitz portfolio. For factor model covariance, the BL model also slightly performs better than the benchmark portfolio. Therefore, we can see that incorporating investors' views into the portfolio optimization will yield higher returns across all risk measures, and beat the equal-weighted benchmark portfolio.

The Table 2 reports the rate of return, Sharpe ratio and standard deviation of portfolios under different risk measures, with and without the implementation of the Black-Litterman model. Consistently throughout all three risk measures, the portfolios perform better with the BL method with higher annualized rates of returns, higher Sharpe ratio, and lower standard deviations.

For CVaR, the Sharpe ratio of the CVaR portfolio raises from 1.4414 to 1.6648, while the annualized standard deviation decreases from 0.7292 to 0.7269 after using the BL method. This implies that the CVaR portfolio constructed with the help of the Black-Litterman method enhances the return while slightly lowering the risk.

Table 2: Comparing Performance between Portfolios with and without Using Black-Litterman Method

Panel A			
	Variance	CVaR	Factor
RoR	0.2679	0.2727	0.2759
Std deviation	0.7280	0.7292	0.7294
Sharpe ratio	1.4602	1.4414	1.4428
Panel B			
	Variance	CVaR	Factor
RoR	0.3308	0.3181	0.2874
Std deviation	0.7270	0.7269	0.7238
Sharpe ratio	1.7127	1.6648	1.6870

Panel A reports the results without using the Black-Litterman Method; Panel B reports the three metrics with the BL method. The three metrics are the annualized rate of return, the annualized standard deviation and the annualized Sharpe ratio of portfolios.

The same improvement on performance happens on the covariance matrix portfolio and the factor model covariance matrix portfolio as the returns on these portfolios increases while their associated risks reduce. In all, during our trading period, using Black-Litterman method with analyst forecasts to build market view significantly enhances the performance of our portfolios.

Comparing the performance of portfolios with the three risk measures, without the construction of BL model, factor model covariance portfolio yields the highest rate of return of 0.2759 with a slightly higher standard deviation of 0.7279. The fact that the factor model covariance portfolio performs the best follows our expectation given that we hold the portfolio during 2016-2018. In addition, the outputs are consistent to the results in our first project, which shows that without the BL model variance is the least satisfying risk measure across time periods and CVaR is the best performing metric during the economic downturns.

Under the implementation of the BL method, the performance of the variance portfolio sees the most improvement with respect to RoR and Sharpe Ratio amongst the three risk measures. It is intuitive since variance purely relies on the historical data in contrast to the other two risk measures. Therefore, using Black-Litterman to generate the expected return vector greatly reduces its dependency on historical returns, which often provides unstable inputs to the Markowitz optimization model. Furthermore, variance is the risk measure that has the highest rate of return of 0.3308 and a Sharpe ratio of 1.6870 with the lowest standard deviation of 0.7270. Therefore, if an investor has specific perceptions of the markets, variance should be preferred among the three risk measures.

8 Conclusion

The Black-Litterman model provides a powerful tool for investors who possess their own views and has an advantage over many traditional asset allocation models. The classical Markowitz portfolio model is sensitive to inputs, and using historical returns as the mean vector tends to yield unstable portfolio allocations. In addition, historical returns are not accurate indicators of how an asset will perform in the future, especially if the asset is very volatile. Driven to examine the effectiveness of the BL model, we select companies in the IT sector in S&P 500 index and cross-examine the performance of the models with different risk measures.

Based on the parameter-tuning outputs over the period of 2010-2014, we found that the results are worse when applying an unrestricted model. A possible explanation is that the portfolio holds more aggressive long and short positions in certain stocks, and hence we have a more volatile and risky portfolio. Therefore, given the tuning outputs, we shed light on the fact that the classical Markowitz model yields extreme weights that are not implementable and practical in real life.

After tuning the constraints and parameters over the period of 2010-2014, we implement our constructed model over for 2014-2018 and provide empirical evidence that incorporating investors views into market views can significantly enhance the portfolio returns across the three risk measures. Furthermore, by comparing the performance among the variance, expected shortfall and factor model covariance matrix portfolios, the model with variance as the risk measure yields the highest Sharpe ratio, largest returns, and lowest standard deviation, which suggests the outperformance of variance as a risk measure under the implementation of the Black-Litterman model.

Our empirical analyses yield similar results to other research papers. Becker and Gürtler (2010) found that all applications of Black-Litterman had achieved good rankings in all performance measures. Bessler et al. (2017) showed that the Black-Litterman model outperformed minimum variance model and naively diversified portfolios, particularly during recessionary periods. Moreover, Zou and Song (2011) proved that Black-Litterman and resampling techniques help generate better allocations as compared to the traditional Markowitz method.

When conducting literature research, we found that a study used analysts' forecasts on dividends to build views based on the dividend discount model (Becker and Gürtler, 2010). They obtained the data from Thomson Financial Datastream, but unfortunately, we did not find the information available on Bloomberg; they also generated views using Monte-Carlo simulation for comparison. Hence, one potential extension for the paper is to use dividends and Monte-Carlo simulation to obtain investors' views and then compare the effectiveness of the different approaches to generate views.

Since our paper mainly focuses on the companies listed in the S&P 500 IT sector and they tend to have a high level of co-movement, it would be interesting to examine the portfolio performance in different sectors as well. Thus, another extension to our project is the inclusion of a broader range of stocks in different and longer time-horizon. Also, examining international and more geographically dispersed stock markets can help enhance the understanding and implementation of the BL model.

For the application of the Black-Litterman model, investors' views on the expected returns are essential for balancing market weights. Since analysts do not make their views and forecasts in the way BL model expects, our paper contributes to the quantitative implementation of the BL model by generating views based on analysts' recommendations and the corresponding one-year target price. The confidence in specified views is determined by the number of analysts' views and the disparity in the recommendations of each asset.

Based on our empirical analyses, the Black-Litterman model avoids the extreme corner solutions that are suggested by the Markowitz approach and provides the investors with more diversified asset allocation. Therefore, we confirm the results from previous literature and provide additional evidence that the BL model is more robust towards unstable inputs to the Markowitz model with respect to the three risk measures. This suggests that the Black-Litterman model should be more widely accepted and applied in the financial industry for portfolio management.

References

Becker, F., and Gürtler, M. (2010). "Quantitative forecast model for the application of the Black-Litterman approach." In *Paris December 2009 Finance International Meeting*.

Bessler, W., Opfer, H., and Wolff, D. (2017). "Multi-asset portfolio optimization and out-of-sample performance: an evaluation of Black-Litterman, mean-variance, and naïve diversification approaches." *The European Journal of Finance*, *23*(1), 1–30.

Best, M. J., and Grauer, R. R. (1991). "On the sensitivity of mean-variance-efficient portfolios to changes in asset means: some analytical and computational results." *The review of Financial Studies*, *4*(2), 315–342.

Black, F., and Litterman, R. (1992). "Global portfolio optimization." *Financial Analysts Journal*, *48*(5), 28–43.

Chopra, V. K., and Ziemba, W. T. (2013). "The effect of errors in means, variances, and covariances on optimal portfolio choice." In *Handbook of the Fundamentals of Financial Decision Making: Part I*, 365–373, World Scientific.

Fabozzi, F. J., Focardi, S. M., and Kolm, P. N. (2006). "Incorporating trading strategies in the Black-Litterman framework." *The Journal of Trading*, *1*(2), 28–37.

He, G., and Litterman, R. (1999). "The intuition behind Black-Litterman model portfolios." *Available at SSRN 334304*.

Jones, R., Lim, T., and Zangari, P. J. (2007). "The Black-Litterman model for structured equity portfolios." *Journal of Portfolio Management*, *33*(2), 24.

Lee, W. (2000). *Theory and methodology of tactical asset allocation*, vol. 65. John Wiley & Sons.

Mankert, C. (2006). *The Black-Litterman Model:*

mathematical and behavioral finance approaches towards its use in practice. Ph.D. thesis, KTH.

Martellini, L., and Ziemann, V. (2007). "Extending Black-Litterman analysis beyond the mean-variance framework." *Journal of Portfolio Management*, *33*(4), 33.

Meucci, A. (2006). "Beyond Black-Litterman in practice: A five-step recipe to input views on non-normal markets." *Available at SSRN 872577*.

Meucci, A. (2009). *Risk and asset allocation*. Springer Science & Business Media.

Meucci, A. (2010). "Black–litterman approach." *Encyclopedia of Quantitative Finance*.

Satchell, S., and Scowcroft, A. (2000). "A demystification of the Black-Litterman model: Managing quantitative and traditional portfolio construction." *Journal of Asset Management*, *1*(2), 138–150.

Zou, Q., and Song, X. (2011). *Implementing The Black-Litterman Model with Resampling: A Typical Investment Portfolio With Hedge Funds*. Ph.D. thesis.